# Computer simulation of laser beam welding for technological applications.

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## Abstract

The article describe a computer model of laser welding with deep penetration base on unification of solution heat- mass transfer problem and problem of irradiation propagation in cavity in approximation of geometrical optics.

## Introduction

High complexity of laser welding make difficulty in choice of process regime by using of technological experiments. Replacement experiments by computer simulating is impossible without identical mathematical model. Such model is needed for equipment designers and at last it can be used as intellect filling of CAD CAM.

Let's consider the requirements for the model, following from its possible applications:

1. The model is to have quite high accuracy and to be so good as accuracy of equipment working parameters establishing not worse than 10 per cent.
2. The calculation time due to technologist requirements is to be limited by seconds.
3. The software must be done for computer technique of technologists (e.g. PC).

Now let's look on the modelling subject. The influence of the radiation flux with density more than $10^4 - 10^5$ W/sm$^2$ results to a narrow and deep cavity forming in the metal [1]. This cavity is surrounded by melt and filled with irradiated material steams, running out the cavity [2]. Vapour pressure together with the reactive return force arising from evaporation counteract the surface stretch forces and keep the cavity open [3]. Influence of the high density radiation on the metal is a complex synergetic process, whose final result determines by joint influence of all partial processes, among which we can select heat, hydrodynamic, gas-dynamic, optic and kinetic ones. Complete mathematical positing of all enumerated problems consists more than 10 differential equations in partial derivations, moreover part of equations and part of limit conditions are not linear. Solving of such system by numerical methods is without of the modern computer possibilities, therefore it should be chosen such method, which give the possibility to obtain solutions of individual problems with their following co-ordination as [3,4].

## Absorbed power distribution over cavity surface.

With laser beam welding with deep penetration the falling radiation distribution problem forms from two parts. The first is connected with determination of laser irradiation absorption coefficients in the temperature interval near the boiling point and in the meeting angle interval ($1^{\circ}$ - $10^{\circ}$). The second part is determination of absorbed power distribution on the cavity surface which connected with its form considering laser rereflection from the walls.
In present model we assume:

- the laser irradiation absorption and reflection coefficients are corresponds with Hagen-Rubens equation and its angle dependencies are given by Fresnel's formulas,
- the incident power distribution calculated separately for parallel and perpendicular polarisation and then summarise this contributions,

Existence of laser rays local reflection from the cavity surface leads to the redistribution of escaping power in depth and perimeter of the cavity. Both the initial radiation flux and flux of radiation reflected from the surface in other points falls on each point of cavity surface. With using geometrical optics approximation is possible to get expression for absorbed power distribution and for both polarization after i reflections:

$$q_{i,II,\perp}(z,\varphi) = q_{i-1,II,\perp}(z';\pi-\varphi)\frac{a(z')}{a}\left(1-R_{II,\perp}\left(\beta(z')-2\frac{da}{dz}\right)\right)\theta\left(\beta(z')-\frac{a}{z}\right),$$

where: $q_0 = Q_0 \exp\{-k(z)(a^2 + \Delta^2)\} \exp\{2a\Delta k(z)\cos\varphi\}$ ,

$Q_0$ - is radiation power density on laser beam axis;

k(z) - is laser beam concentration coefficient depending on the longitudinal coordinate "z" because of focusing;
z - is coordinate being marked off from the sample surface,
z' - is z-coordinate of reflected area,
a - is the cavity radius;
$\Delta$ - is laser beam axis displacement from the cavity axis;
$\varphi$ - is a polar angle.

Taking into account three first reflections for total absorbed power distribution we get:

$$q\left(z,\varphi,a(z),\frac{da}{dz}\right) = \sum_{i=0}^{3}\left(q_{iII} + q_{i\perp}\right).$$

Here $\beta$ - is a meeting angle; $\eta(\beta)$ - is reflection coefficient. The distribution of total absorbed power along cavity surface can be calculated only with calculation of cavity shape in complex model.

## The heat and mass transfer in acting of concentrated energy fluxes on the metal

Heat and mass transfer in laser welding with deep penetration are complicated in the theoretical description. In considering this problem we have to deal with the heat transfer and the mass transfer, proceeding in extremely close connection with each other. So, the solution of the hydrodynamic problem determines coefficient values in the heat transfer equation, and the solution of the heat problem determines boundary conditions for the hydrodynamics equation. In mathematics the problem is also complicated by "physical" non-linear present, there are viscosity, thermal conductivity and temperature conductivity dependencies on temperature, and by "geometrical" non-linear present, that is present of the "free" boundaries of rated areas, being determined right from the solution. At the same time there is a number of factors that makes a solving of this problem possible in spite of enumerated difficulties. The first of them is connected with deep penetration present. In this case a space scale along the cavity axis "H" is much more than typical transverse scale "a". It allow us estimate the derivatives of determining functions (one of temperature is "T" and one of melt speed is "V"):

$$\frac{\partial T}{\partial z} << \frac{\partial T}{\partial x},\ \frac{\partial T}{\partial y}\ ;\ \frac{\partial V}{\partial z} << \frac{\partial V}{\partial x},\ \frac{\partial V}{\partial y}$$

That's why we can reduce the problem to two-dimensional one.

We consider the hydrodynamic problem first. Distribution of velocities in the melt are described by Navye-Stoks' equation:

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \nabla\right)\vec{V} = -\nabla \frac{p}{\rho} + \nu \Delta \vec{V}; \quad \nu = \frac{\eta}{\rho}$$

We'll consider stationary situation  when a velocity field doesn't depend on time. Due to Reynolds' numbers are extremely big with typical parameters for laser beam welding with deep penetration (10 -1000), we can ignore the viscosity part in first approximating. In this case liquid flow can be considered as a potential one with potential $\varphi_V$, answering the Laplas' equation: $\Delta\varphi_V = 0$. Let's formulate boundary conditions for the velocity potential. On the cavity surface is to be a condition of surface impenetrability for a liquid flux (because the evaporation flux is extremely small), and at the melting front is to be continuity of the velocity part, that is normal to the boundary:

$$\left.\frac{\partial \varphi_V}{\partial \vec{n}}\right|_{r=a} = 0\ ; \qquad \left.\frac{\partial \varphi_V}{\partial \vec{N}}\right|_{G} = -\vec{V}_0 \cdot \vec{N} \qquad ,$$

where “G” is a melting front, $\vec{N}$ is external normal vector for melting front.
The solution can be obtained by combination of values division and mapping by complex argument function of cross-section, that is perpendicular to the cavity axis, on a concentric ring:

$$\varphi_V = -V_0 \cdot \sum_{n=1}^{\infty} \frac{b_n \cdot R^{2n} - b_{-n}}{R^{2n} - a^{2n}} \cdot (r^n + \frac{a^{2n}}{r^n}) \cdot \cos(n \cdot \varphi_r) \quad ,$$

where r, $\varphi$ - are polar coordinates, $b_n$ is a coefficient of mapping function expansion in row of Loran: Z=$\sum_{n=-\infty}^{\infty} b_n \cdot \tau^n$.

Because of it is necessary at the same time to solve the heat problem determining a melting front, that was done with iterations. First liquid velocities field is determined without considering the melting boundary then temperature field and a melting boundary correspondingly are determined with regard to the velocities field, then a new velocities field is determined with considering the melting boundary and the process is repeated. Consideration of melt viscosity influence lead to the difference of velocity field from ones given by this expressions nearly the cavity surface and the melting boundary. Boundary layers are formed in these areas. The viscous forces result in the appearance of eddy trace with angle wide $\theta \approx 3\sqrt{\pi}/\sqrt{\mathrm{Re}}$ in the rear part of the welding bath [6]. Since typical space scales of boundary layers and eddy trace are great less than ones for main flux, and velocity scales coincide, so Peklet’s numbers are great smaller for the boundary layers and eddy trace than for the basic flux. Thus in the first approximation we can neglect by effect of boundary layers and eddy trace a temperature field, and so it can be confined our consideration to the convective heat transfer influence, which is concerned the basic liquid flux.

The heat field is determined by the heat transfer equation with obvious boundary conditions:

$$\vec{V}(\vec{r}) \cdot \nabla T = \chi \Delta T \ ; \qquad T|_{r\to\infty} \to T_0; \quad \left.\frac{\partial T}{\partial \vec{n}}\right|_g = -\frac{q(\varphi)}{\lambda} \quad ,$$

where g - is the cavity boundary;
$\vec{n}$ - is external perpendicular to this boundary;
$q(\varphi)$ - is density distribution of the heat flux falling on the cavity surface.

For solving this problem it is convenient to pass from “physical” plane (x,y) to the plane of complex flow potential $\Phi = \varphi + i \cdot \psi$, where $\varphi, \psi$ is a flow potential and flow function.

Solution can be obtained by Fourier’s integral transposition. After transformations we get:

$$T(\zeta,\eta) = \frac{a}{\pi\lambda}\exp\{Pe\zeta\}\int_{-1}^{1}\frac{q(-\zeta')\exp\{-Pe\zeta'\}}{\sqrt{1 - \zeta'^2}}K_0(Pe\sqrt{\eta^2 + (\zeta - \zeta')^2})\ d\zeta'$$

$$\begin{cases} \zeta = -\frac{1}{2}\left(\frac{r}{a} + \frac{a}{r}\right)\cos\varphi \\ \eta = -\frac{1}{2}\left(\frac{r}{a} - \frac{a}{r}\right)\sin\varphi \end{cases}.$$

This expression analysis shows that the consideration of the convective heat transfer changes isotherm form displacing the maximum welding bath depth down along the flux and making the welding bath longer. This solution is easy to improve with using the perturbation method to take into account physical coefficients dependencies from temperature and the heat of the melting-crystallisation phase transition.

## The calculation of melting zone parameters in laser beam welding

The combination of models of physical processes considered before allows to calculate melting zone and cavity shape, cavity surface temperature and other necessary values by the next algorithm: we put falling power value considering reflection into the expression for the heat field and suppose r = a, that corresponds to the cavity surface. Then we obtain an equation connecting the surface temperature with the cavity radius "a" and its walls slope angles. The second equation of the mathematical model is a force balance condition between vapour pressure in the cavity and the Laplas pressure $p = \sigma/a$ compressing the cavity. Analysis of evaporation in beam acting on a metal shows the vapours condition nearly saturating [7]. That's why we can use the expression for the equilibrium pressure of the saturated vapours for the vapours pressure to connect with the surface temperature:

$$p = A\exp\left(-B/T\right) ,$$

where A, B - are table constants.

To get more precise equation for vapour pressure - surface temperature dependence it is necessary to solve the gas-dynamic task. For welding with deep penetration it is possible to use one-dimensional approximation for this purpose. The complete formulating of this problem contain three equations of continuity: for mass flux, movement flux and energy flux [5]. It should take "T" as an averaged over the perimeter value of surface temperature with fixed co-ordinate "z". Excluding temperature from these two equations we get differential equation for the description of the cavity surface shape:

$$f\left(a, \frac{da}{dz}\right) = 0 \quad .$$

Let's put $\Delta z$ as a step by co-ordinate "z" and number the steps. Then $da/dz = (a_{k+1} - a_k)/\Delta z$ for the cavity radius on k-th step we take its mean value: $a = (a_k + a_{k+1})/2$ .

Then after discretization we obtain:

$$f(a_k, a_{k+1}) = 0 \quad .$$

If $a_0$ is known, we can construct the cavity profile, solving the equation numerically by either way. For determining $a_0$ it should be solved the previous equation relatively "a" with given "da/dz", that can be done. In solving last equation it should be determined both the cavity radius and a power $\Delta Q$ being absorbed by layer with thickness $\Delta z$ for every step. The calculation should be stopped in achieving the condition:

$$\sum \Delta Q_k = W$$

where W - is full power of the incident radiation.
The values of $\Delta Q_k$ are the density distribution of equivalent linear heat source and they can be used in calculating of the heat fields. Example of calculations is shown on fig. 1 for welding of Al alloy with 4% Zn and 1% Mg. Beam power distribution for TEM $_{00}$ mode shown on figure for the depth value correspondingly 0, 0.27 and 0.55 cm.

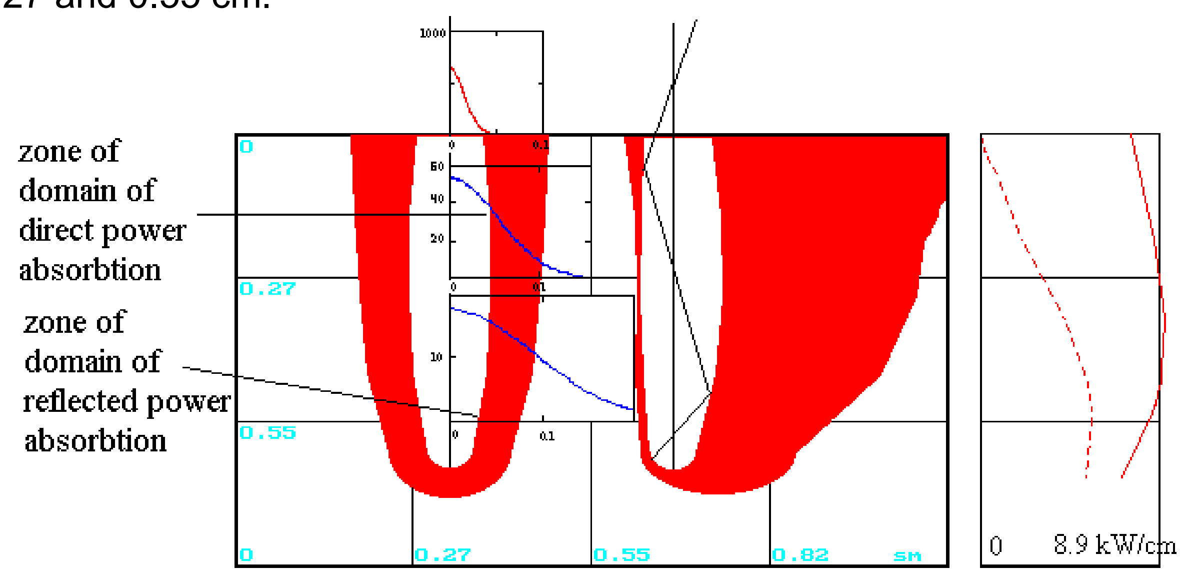


fig.1

Solid line - total absorbed power distribution, dot line - distribution of absorbed power that was reflected above, velocity of feeding is 5 cm/c, beam power is 4.9

kW, focus position is 0.1 cm above sample surface, focusing length is 15 cm, Fresnel number is 4.7, focal radius (with 86% of total power inside) is 0.0116 cm.

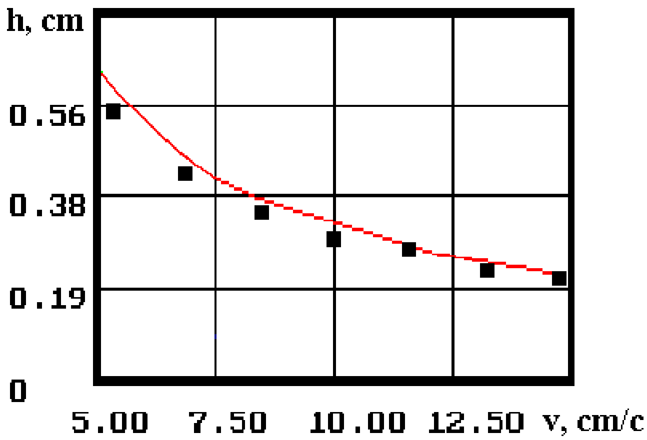


Fig.2 shown comparison between calculated results and experimental ones by Joachim Berkmanns, ILT, Aachen, for penetration depth dependence from feeding velo-city. The working parameters are the same as for previous picture.

Differently from existent models of laser welding [3,4 ] the model described above take into account simultaneously energy and pressure balance on the cavity walls, convective heat flux in melt pool and polarization of incident power. PC - software designed on the base of this model allow calculate the cavity and melting pool shape for one number of regime parameters fast then 30 seconds on PC-486DX4.

## References

1. S.I.Anisimov, Ya.A.Imas, G.S.Romanov, Y.V.Hodyko. Action of high power radiation upon the metals. Moscow, “Nauka”, 1970, 272 p. (rus).
2. J.Dowden, M.Davis, P.Kapadia, Some aspects of the fluid dynamics of laser welding. J.Fluid Mech, 1983, V. 126, p. 123 - 146.
3. M.Beck, P.Berger, M.Hugel. Modelling of keyhole-melt interaction in laser deep penetration welding. ECLAT’92, p. 693 - 698.
4. A.Kaplan. A model of deep penetration laser welding based on calculation of the keyhole profile. J.Phys.D: Appl.Phys., 27 (1994), p. 1805 - 1814.
5. Turichin G.A. Physical bases of the joint formation process in EBW with deep penetration. The dissertation. Leningrad, LPI, 1990, 221 p. (rus).
6. Mitkevich E.A., Turichin G.A., Illyin S.A. Influence of hydrodynamic phenomena on the temperature distribution in EBW. // Thesises of reports of X conference “EBW”, Leningrad, 1988, p. 14 - 15. (rus).
7. Pavlukevich N.V., Gorelik G.E., Levdanski V.V. Physical kinetics and transfer processes in phase transforming. Minsk: “Nauka i Technika”, 1980, 208 p. (rus).