THEORY OF LASER RESONATORS AND OF THE BEAM DIVERGENCE

A Review

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My intimate acquaintance with lasers dates as far back as 35 years: indeed, in 1962 I succeeded in achieving generation in the first Russian lasers with cryogenic cooling. During the subsequent years, laser technology has made amazing progress; in particular, radiation brightness increased several orders of magnitude, current-day lasing spectra are immeasurably more narrow than those of the first lasers, and the beam divergence problem has gone a considerable way toward its solution. And while one should not certainly underestimate the importance of the laborious work on making gain media and their excitation systems more efficient, nevertheless improvement of the laser resonators has contributed to a very large extent to this progress.

The resonator is known to be a major component of the laser. It is here that the lasing radiation whose specific properties distinguish the laser from among the other sources of light is formed. And it is on the resonator problem, and on the problem of the laser beam divergence which is closely related to it, that I am going to dwell in my communication.

We shall start with a bit of history. The cornerstone of the resonator theory in its present form was laid by the brilliant work of Fox and Li [1]. This work demonstrated for the first time the existence of modes in open resonators and revealed the main properties of these modes for a few simplest cases. The introduction of the concept of diffraction losses has proved to be essential; it has turned out to be much more useful as applied to laser resonators than their quality factor and has almost completely superseded it.

Reading the work of Fox and Li today still rouses admiration; its impact on the subsequent developments in this area has been so profound that even the small errors overlooked by the authors (see p. 81 in my book [2]) is still present in many textbooks on quantum electronics.

This work was followed by papers of Boyd, Gordon, and Kogelnik [3, 4], who made a more general analysis of open resonators consisting of two spherical mirrors with arbitrary radii of curvature, and proposed a classification of such resonators according to the corresponding diffraction losses.

The works of Collins and of Kogelnik and Li published in the mid-60s [5-7] were the next major step forward. They developed general methods to analyze devices made up of an arbitrary number of simple optical components arranged on a common straight line. These methods provided the possibility of reducing such systems to equivalent two-mirror arrangements.

One of the most essential works which made a significant impact on subsequent progress in quantum electronics was a paper [8] published at about the same time by Professor Siegman who has honored this workshop by his presence. It attracted general interest to the so-called unstable resonators, which until that time seemed to be useless. As a result of the subsequent theoretical works of Siegman and of an extensive series of our theoretical and experimental studies (see [2], sections 3.5 and 5.1.4), it is the resonators of this class that have become employed on an increasingly broader scale in high-power lasers with a small beam divergence. We have proposed, studied, and implemented specific arrangements based on these resonators, which are capable of providing presently an answer to a variety of problems in quantum electronics. I have in mind multipass amplifiers with gigantic gain in one stage, field rotation resonators with compact output aperture, resonators with super-high stability of emission direction etc ([2], sections 5.3, 5.4).

Thousands of works dealing with resonators for diverse applications and designed to operate in various conditions have appeared hence. An analysis of the properties of even empty resonators turns out sometimes to be very cumbersome. The task becomes progressively more complex when a resonator contains a gain medium, and one has to take into account the various nonlinear processes involved. Note also that the geometry of radiation interaction with the excited medium is in many cases far from simple. For all these reasons, making now even a cursory review of all these studies would be a problem of daunting complexity indeed, and therefore we shall restrict ourselves only to a broad-brush account of the aspects we believe most interesting.

In nearly all the above mentioned works, the various methods of analysis were supported by the results of numerical calculations, or were based totally on such results. The explosive growth in popularity of personal computers makes numerical calculations increasingly more important in all walks of our life, including resonator design. Let us therefore devote a few words to the methods used in such calculations.

Until now the most popular method is the one used for the first time in that very famous paper of Fox and Li [1]. It is based on considering the evolution of a monochromatic light beam in its multiple transits around the resonator.

This procedure is essentially an iterative technique to find the eigenfunctions of the corresponding integral equation. For over more than one decade, this iterative procedure has been subjected to only insignificant improvements. In particular, one has succeeded in increasing somewhat the number of the lowest modes amenable to calculation by this method; it has been found that the iterative approach permits one to include a nonlinear gain medium into the analysis. At the same time using in each iteration step the cumbersome calculational process based on the Huygens-Fresnel principle resulted quite frequently in unjustifiably long computing times.

A large step forward was made only in the mid-70s, when Professor Siegman together with Sziklas proposed the so-called fast Fourier transform [9, 10]. This method permitted one in many important cases to cut down dramatically on the volume of numerical calculations and, thus, to broaden the range of the resonator parameters which can be calculated numerically.

A highly original approach was proposed by L. A. Vainshtein and described in his monograph [11]. This approach permitted Vainshtein to explain some of the processes which occur in resonators but did not at the time obtain satisfactory interpretation, and to obtain relatively simple expressions for a number of important cases which did not yield to analysis by other methods until then.

The approach of Vainshtein, which is based on taking into account diffractive reflection from resonator edge, was quite popular in the 60s - 70s. It was found subsequently that the part played by this process is mostly not large, and this method is employed presently only in certain particular cases.

The analysis of the evolution of coherent light beams based on using the concept of point eikonal, which is essentially the optical path, has enjoyed a different fate. This method is popular in classical optics and, when applied to the theory of resonators, was extended considerably and is presently used both in studies of an analytical character and in numerical calculations.

This approach was initiated by the paper of Collins [5] published in 1964, which has already been mentioned by us. In this very first work, Collins succeeded in establishing fairly general properties of resonators with simple astigmatism (such optical systems are called frequently orthogonal).

The work of Collins did not, however, contain useful recommendations for calculation of the eikonal, and therefore subsequent studies proceeded primarily in the direction delineated by the classical works of Kogelnik and Li [6, 7], likewise mentioned already. The ray matrix formalism proposed by them yielded an extremely simple expression, called sometimes the ABCD relation, which describes the behavior of Gaussian and similar beams, generated primarily by lasers.

Sometime between the 60s and the 70s it was found that matrix formalism permits one not only to use the ABCD relation but to calculate the eikonal itself (see, for example,[12]; thus these two methods have become intimately connected. Their synthesis leads to universal integral relations, which make possible finding the field distribution at the output of multi-component optical systems from the given distribution at their input.

During the many subsequent years this method could be applied, however, only to a comparatively narrow class of optical systems possessing two mutually perpendicular planes of symmetry. Only in the late 80s, I together with Bekshaev have succeeded in extending this method to systems with arbitrary astigmatism and misalignments, which contain, besides optical components which are comparatively easy to take into account, inclined ellipsoidal interfaces, diffraction gratings, and sections of a medium with a refractive index whose dependence on transverse coordinates is described by a complex second-order polynomial [13, 14]. The results obtained in these studies are summed up in Appendix to my book [2]. We are presently completing, in cooperation with scientists from Aachen, development of computer programs based on these methods.

Besides the above mentioned methods or in combination with them, there are many others adopted from quantum mechanics, automatic control theory etc. We are not going to dwell on them; it should now be clear that the theory of laser resonators has long become an independent branch of quantum electronics. People specializing in this area, just as in any other, possess a large volume of specific knowledge and have accumulated immense practical experience in resonator design. This specifically relates to those of them who participated in development of lasers of various types. Among them are my former students delivering a number of review lectures at this workshop, namely representing Russia Sherstobitov, Orlova, Dimakov and presently Canadian Anikichev. I should mention that the knowledge of the technical solutions proposed in diverse situations permits one to master virtually all techniques used to improve laser optics. This provides a considerable edge over other designers of laser technology who quite frequently have dealt only with one laser type and may not be aware of the useful solutions reached for lasers of another kind.

Nevertheless, one frequently meets in practice with an underestimation of the importance of this experience and knowledge. Therefore I am going to explain now what is the use of people specializing in optical resonators and, on a broader scale, in methods of solving the problem of the quality of laser radiation, a problem of paramount importance indeed.

I immediately point out the expediency of their participation in the very early stages of a new laser project. This is sometimes neglected for one of the two following reasons: specialists in methods of excitation of gain media either believe that their own knowledge in resonators is more than sufficient or are of the opinion that one has first to make a laser, and the beam divergence problem can be shelved until a later time. It is not hard to guess what such an approach can lead to. I know of more than one case where time-consuming work on development of a laser of a new type proceeded for a long time without invoking the help of resonator specialists in a direction which, from the standpoint of spatial characteristics of radiation, was totally a blind alley. And after the laser has been built, and it turns out that its operation is unsatisfactory, the people in charge come to a sudden realization that something is wrong, but the train has already departed, as we say.

As an illustration, I could tell here about some very expensive projects on development of high-power liquid-medium lasers started in our country. At the same time it would not have been difficult for resonator specialists, acquainted at least superficially with thermo-optical properties of liquids, to draw a conclusion on total fruitlessness of such ventures.

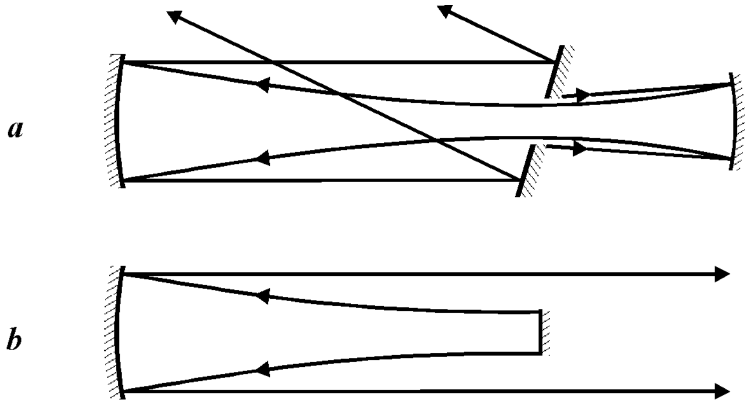
After the general direction of work on the project has already been chosen, one starts with developing the optical arrangement of the corresponding laser, which sometimes requires searching for novel solutions; we shall see at this workshop that the process of generation of these new solutions has not come to an end. As a rule, such novel solutions will require experimental verification, and professional experience and expertise in resonator development permits one sometimes to carry out this verification by straightforward and cheap means. For instance, we managed once in avoiding truly colossal expenses by simulating resonators for giant chemical lasers using the simplest neodymium-glass laser.

Although at first glance the laser resonator appears to be a simple device of which everybody knows everything, it is by far not every laser-technology designer that is acquainted with its subtle features.

This relates especially to the arrangements that have appeared quite recently. I shall permit myself a few fairly recent examples.

In the early 80s, Italian researchers proposed the so-called unstable resonator with spatial filtration of radiation (SFUR) (see its schematic in Fig. 1,a). In contrast to conventional unstable arrangements, the output distribution of this laser is close to Gaussian. True, at the center there is an area unfilled by radiation, but if the medium provides a high gain this area is small, and it does not affect strongly the far-zone distribution.

Figure 1

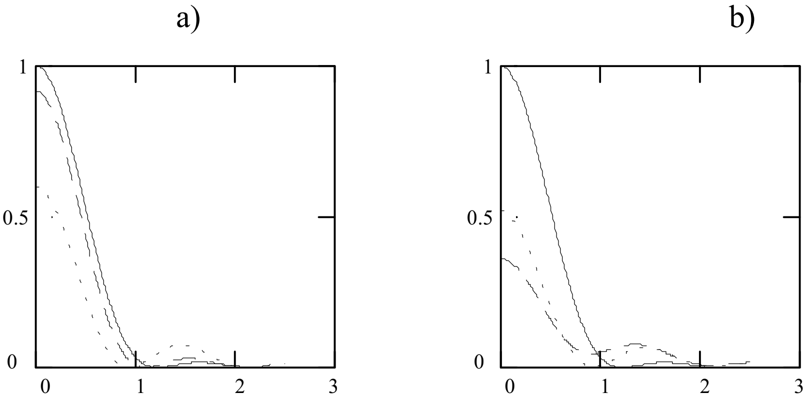


Many researchers are acquainted with this arrangement and sometime use it, but much less known is the following: our analysis [15] showed that the right-hand part of this resonator can be replaced by one plane mirror (Fig. 1,b). The new resonator, while providing the same lasing parameters, is not only simpler in design but at the same time is more convenient in that the radiation is led out here along the optical axis rather than off it.

A few years ago a group of Japanese researchers started to employ unstable resonators with a semitransparent output mirror. The idea behind this arrangement is that adding to an annular cross-section beam at least a weak beam with an appropriate phase from the central zone could increase substantially the fraction of the energy confined within the central spot in the angular distribution.

This approach is illustrated by Fig. 2, a. The upper (solid) curve shows the angular distribution of radiation for an ideal plane-wavefront emitter with uniform distribution over a circular output aperture, the lower (dot) one relates to a conventional unstable resonator with 60%-losses, and the third (dash), to unstable resonators which have output mirrors with a finite transmission coefficient T=0.3 but the same losses (for which purpose one has to reduce the coefficient of magnification M). We readily see that in this idealized case of no intracavity aberrations this approach can turn out quite useful.

Figure 2



This example gives us grounds to voice the following consideration. The authors proposing a new arrangement try usually to stress all its merits, however small, while passing over in silence its drawbacks. But the specialist using various designs in practice must know these shortcomings. In this particular case, this arrangement has a serious drawback which our analysis has immediately revealed, namely, it is highly sensitive to intracavity aberrations. This is readily seen from Fig. 2, b, where the upper curve is the same as on Fig 2, a, and two others relating to the same resonator designs as Fig. 2, a, but in the presence of spherical wave aberrations of only $\lambda/8$ in single transit though an active medium. In these conditions resonators with partially transmitting mirrors turn out to be not better but worse than the conventional designs.

Rather than multiplying examples of such kind, we shall devote a few words to the beam divergence problem, which is intimately connected with the resonators.

The factors on which the angular divergence depends are well known, as is well known also the large variety of means by which one can try to approach its diffraction-limited value. While introducing unstable resonators permitted one to make considerable progress on this way, there still exist the limitations associated with inhomogeneities of the gain media. Attempts at overcoming these limitations by means of the so-called wavefront conjugation have been made with varying success for over a quarter of a century. The history of this method goes as far back as the mid-60s, when Kogelnik proposed it to improve observation systems; in 1971, I proposed a similar technique to correct the wavefront of high-power laser emitters [16] (here one has in mind the very popular scheme of two-transit laser amplifier with intermediate wavefront conjugation). .

An analysis of all these problems on this workshop being impossible, I shall talk only about what one should strive for. I have in mind the radiation field distributions which are most favorable from the applications standpoint, an aspect which is connected intimately with the criteria to be used in comparing various emitters of the same power from view-point of their angular distribution.

In our country the angular divergence of laser radiation was most frequently characterized by one of two parameters, namely, the divergence at 0.5 intensity level and that at 0.5 (sometimes 80%) energy level. The first of them is actually the width of the central maximum in the angular distribution which is measured at the level corresponding to one half the maximum intensity, and the second, the angular width of the cone confining one half (or 80%) of the total radiation flux.

Any systematic analysis of the shape of the angular distribution radiated by various coherent sources showed, however, that attempts at comparing the practical significance of sources in the value of only one of these two parameters can easily lead to an erroneous conclusion. Take, for instance, the angular distributions produced in diffraction of a plane wave from a round hole and from rings with the same outer diameter (the latter shape is characteristic of conventional unstable resonators). Calculations yield the following results: the angular divergence of the radiation emitted by round hole measured at the 0.5 intensity level or at the 0.5 energy level is nearly the same; but when we proceed to annular emitters, the first value decreases and the second quickly increases. For instance, if inner diameter of ring is equal to 80% of outer diameter, these two parameters differ more than 6 times.

The general conclusion that any attempt at characterizing the divergence in any situation with only one parameter seemed to me fairly obvious (see, foe example, my books). Nevertheless, in the late 80s Western specialists in laser technology accepted the so-called "beam quality" M^2 as such a universal parameter.

This parameter is based on calculation of second-order moments of near- and far-field intensity distributions. The theory of beam moment evolution in the course of propagation is in itself quite elegant, and Professor Siegman played a prominent part in developing and spreading this method. A certain contribution to this area is due to me with Bekshaev [17]; indeed, whereas before our paper the moment evolution laws were known only for beams of comparatively simple shape propagating through orthogonal optical systems, we solved the problem for arbitrary beams and for systems of a much broader class.

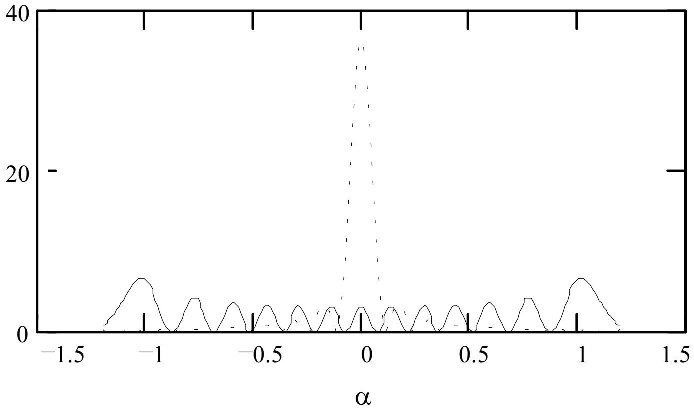
The definition itself of the beam quality in terms of second-order moments implies that the significance of the distribution tails, for which the weighting factor equal to the squared distance from the axis is too large, is here clearly overevaluated. A more careful analysis shows indeed that the beam quality, as a criterion for comparing various sources of coherent radiation in their usefulness, has an inherent deiciency, which do not have the parameters mentioned before. Indeed, the beam divergence of any radiation source of practical significance, measured at any intensity or energy level, has always a definite and finite value. At the same time the beam quality has a finite reasonable value only if fairly rigid conditions imposed on the field distribution shape are satisfied. For many realistic coherent-radiation sources these conditions are formally not met.

Indeed, it can be easily shown that if the dependence of intensity on transverse coordinates has breaks, the second moments of the angular distribution and, hence, the beam quality become infinitely large. Note that such breaks inevitably occur in the case of diffraction from rigid diaphragms, which are always used in optical systems.

While in principle this difficulty can be sidestepped by introducing an apodization factor in a theoretical consideration, or by limiting properly the aperture in far-field distribution measurements, the result will now become dependent on the width and shape of the corresponding transmission function**.**

Because of that, the comparison of the potential of different sources by the magnitude of M^2 can lead to absurd results. The well-known paper of Siegman [18] may serves convincing illustration of this fact. I shall permit myself giving here one more revealing illustration; just as in the paper of Siegman, it relates to the case of phase correction for beams with alternating amplitude distribution. Solid curve in Fig. 3 displays the angular distribution of radiation for a high-order mode of a stable resonator (it is for such beams that this method of correction was originally proposed); after the correction, the angular distribution assumes the shape of the dot curve.

Figure 3



It is certainly obvious that the phase correction has dramatically improved the angular distribution; indeed, the central lobe, which originally contained 4% of the total power, now confines 70%. At the same time the quantity M2 retained its original value of 25 after the correction only because a negligible fraction of the radiation is scattered through large angles.

I think the most universal parameter characterizing an angular distribution in the cases where the output laser aperture does indeed limit the beam cross section and, thus, determines the diffraction-limited divergence, is the aberration factor introduced in my book [19]. It is equal to the ratio of the axial luminous intensity (i.e. intensity at the center of the angular distribution or of the focusing zone) to its maximum possible value at the given output power and cross section of output aperture. It is interesting that above-mentioned maximum possible value of the axial luminous intensity is reached in the case where the wavefront is plane, and the radiation density fills uniformly the aperture rather than has the Gaussian distribution.

For problems associated with illumination of distant objects the adequacy of this criterion is obvious. Besides this case there is a number of other important applications where Gaussian beams are far from optimal.

In connection with all this I do not understand why the manufacturers of industrial lasers show such inclination toward Gaussian beams; indeed, despite the fact that the people with whom I presently work have great experience in the use of lasers in cutting, welding etc., we have not been able to find any weighty arguments in favor of application of pure Gaussian beams in any process.

All this provides support for my opinion, expressed more than once, that it is impossible to characterize the beam quality with an only parameter appropriate for any conceivable case. Beams with different intensity distributions can be found suitable for different applications, and, thus, one will inevitably have to use different beam quality criteria.

In conclusion, I shall permit myself a short and possibly somewhat disagreeable digression. At any conference dealing with a broad scope of problems bearing on resonators and the angular divergence of radiation one invariably hears too many ideas and learns about so many studies which had been reported and discussed in Russian-language literature long ago.

This workshop has not been, alas, an exclusion to the rule. Rather than presenting numerous examples in support of this statement, I shall restrict myself to only one of them. I have in mind the report of Prof. Siegman, whom I hold in a high esteem and to whom, by the way, I am greatly indebted. The concept developed in this report was proposed by me and Anikichev as far back as 1986 [20].

It will not be an overstatement to say that during the recent two decades I have come across ideas and considerations new to me only on a few occasions. There are two reasons for this.

One of them consists in that in the late 60s Russian researchers have found themselves in a particularly favorable situation. At that time, our country was spending huge amounts of money on military purposes; this provided us with a unique possibility to bring together a large group of good specialists, which now already for 30 years have been working exclusively on problems connected with resonators and the angular divergence. This group has been taking part for a long time in practically all large Soviet laser projects and has accumulated a truly enormous experience. We studied comprehensively all problems arising in building resonators for high-power lasers. All results of scientific significance were published in literature; it should be added that rather than limiting ourselves to listing the various methods of solution for the corresponding problem, we expressed our opinion concerning the potential of these solutions. And we can see now that in the vast majority of cases these opinions have turned out to be correct.

The second reason, and not a less significant one, is that nobody in the West appears to read our papers and books, although our magazines and books are published in an English edition as well. One has to admit that the quality of translation is quite often unsatisfactory. This does not, however, relate to my recent book [2] which summed up numerous Russian studies. Translated by my good friend G. P. Skrebtsov, it was published also in the West; and although all reviewers estimated the translation as excellent, and a preface to the book was written by Prof. Siegman himself, nobody reads it either, as this has become obvious at this Workshop!

At the same time if no efforts were spent to understand anew what was already learned before, and no mistakes which were made by other researchers were repeated time and again, our progress would be much faster and much more rational.

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