Resonator and beam divergence problem

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ABSTRACT

The problem of the continuity of knowledge in the field of laser resonators and the beam divergence is considered. Some interesting episodes of the laser history are expounded; one gives a grand attention to works of Russian scientists.

Keywords: laser, resonator, wave, conjugation, mode, gain medium, perturbation, inhomogeneity

1. INTRODUCTION

The title of my report under which it was included by the Organizing Committee into the Conference Program is too general. It would certainly be impossible to cover in half an hour the subjects discussed in slightly more than a cursory way in my bulky book [1] bearing the same title. Therefore I shall restrict myself primarily to the problem of the continuity of knowledge.

The amount of information on laser physics became already long ago so immense that no one researcher would be able to acquaint himself with a sizable part of it. At the same time when planning a specific study one should naturally know what has been done in this area, and to be at least moderately informed of the information that can be found on related subjects. In practice, however, this is quite frequently not so.

For already at least two decades I have been stumbling onto cases where one discovers anew relations which have been known for a long time, or invents devices which are available or even mass produced somewhere. Still more frequently studies made by the trial and error approach are pursued in directions which have already been tried by somebody and found to be blind alleys.

These immense expenses of time and effort are more often than not the result of insufficient knowledge of the experience gained by predecessors. Particularly poor is the knowledge of what has been done by the Russians; Western specialists do not, as a rule, read Russian publications in the language of the original, while the available translations too frequently do not hold water. I know this particularly well from my experience on the Editorial Board of the academic journal “Optika i Spektroskopiya”. In some cases, however, the Board was permitted to choose their own translators, which improved dramatically the translation quality. Eventually, however, the right of translation of this and of many others Russian journals was transferred to another organization, which does not check the adequacy of terminology, and, as a result, it has become virtually impossible to understand the meaning of our papers.

I met one more time with a practically total ignorance of Russian literature on the subject when attending the NATO Conference on Resonators held in Slovak Republic last summer.

All this provides me with an excuse, rather than attempting to cover the problem of laser resonators and of the angular divergence as a whole, to touch on a number of aspects which are interesting from my personal viewpoint and, in doing this, to focus attention primarily on the relevant information that can be found in Russian-language literature.

2. INITIAL STAGE OF OPEN RESONATOR THEORY DEVELOPMENT

One of the most fundamental areas in laser physics is the theory of open resonators. The paper of Fox and Li [2] provided a firm foundation for this theory as it is known presently. It is this study that presented for the first time the integral equation of the open resonator, proposed an iterative technique for its solution which is still used in most of numerical calculations, and introduces the crucial concept of diffraction losses. I shall permit myself here only to comment on a certain illogical point made by Fox and Li in deriving their integral equation, which becomes particularly manifest when lecturing on this subject.

Indeed, the formulation of the Huygens-Fresnel principle used in this study does in no way describe propagation of a light wave in time, but rather relates the distributions of the complex amplitude of stationary light field in different planes but at the same instant of time. Therefore application of this principle to a description of wave evolution in one complete round trip around a resonator should produce a field distribution in the original plane at the same instant of time, which certainly must coincide with the original one and not differ from it in the presence of a constant factor. Besides, one should use for description of the field decaying in time inside an empty resonator the complex rather than real propagation constant. The integral equations in this form, which is more adequate, can be found in studies of L. A. Vainshtein, on which we shall dwell here somewhat later. One can found more detailed consideration of this problem in paper [3].

The work of Fox and LI was followed by fundamental papers of Boyd, Gordon, Kogelnik, Collins, Li [4-8], which essentially completed development of the theory of empty ideal stable resonators.

3. VAINSTEIN'S METHOD

The theory of plane resonators had a different fate. Although it is the plane version that was the first open resonator for which the shape of the lowest modes was calculated [2], the nature of these modes remained unclear for many years to come. As a matter of fact, any wave of a finite cross section contains inclined components, which, as it would seem, should escape after a certain number of round trips from the plane resonator and, thus, destroy the reproducibility of the total field distribution in shape.

The studies of L. A. Vainshtein, which were summed up in his fundamental monograph [9], showed that the mechanism of field confinement inside a plane resonator is essentially different from the one realized in stable systems; this mechanism involves diffractive reflection from the resonator or waveguide edge. This is illustrated by Fig. 1: when a waveguide wave consisting of two slightly inclined light beams transforming into one another strikes the open edge (dashed line) of a waveguide made up of two plane mirrors, the major part of its energy, rather than escaping from the waveguide, is reflected back into it due to some specific diffraction effects.

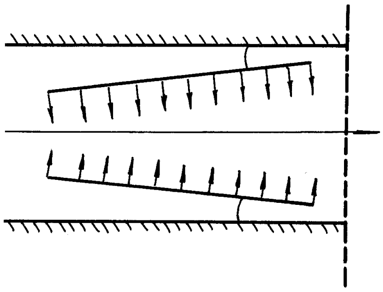

Fig. 1

An analysis of this effect permitted Vainshtein to derive for the first time analytic expressions for field distributions and mode losses in plane resonators. Besides information bearing not only on plane but on empty resonators of other types as well, his book contains many fundamental principles of the general theory of open resonators. For example, one can find there important considerations on the possibility of expansion in sets of open-resonator eigenfunctions; it is shown that the fundamental mode of a confocal resonator made up of mirrors of the same curvature provides optimum transport of ray energy between the apertures of these mirrors, and so on. Permitting myself a slight digression, I shall add that we have succeeded subsequently in obtaining a remarkably simple substantiation for the principle of energy transport [10], as well as in proving that the eigenfunctions of all resonators consisting of mirrors of finite dimensions, except for the confocal one, do not form a complete orthogonal set [1].

Russian literature abounds in theoretical works using the method of Vainshtein. Particularly successful in this were V. V. Lyubimov and I. B. Orlova. They succeeded even in explaining the intricate behavior of the modes of ideal unstable sharp-edge resonators by invoking already not one but several waveguide waves scattered by edge effects toward the resonator axis.

4. ABOUT NON-IDEAL RESONATORS WITH ACTIVE MEDIUM

We shall continue to discuss unstable resonators somewhat later. As for other Russian studies of that period in the area of resonators, they focused major attention, in contrast to Western papers, on the properties of nonideal resonators, the role of an inhomogeneous gain medium they contain, and so on. This was apparently due to the fact that the people working with resonators in Russia at that time were primarily specialists on solid-state lasers, for whom these problems are of crucial importance.

I would like to mention that I was possible the first to establish the dependence of the optimum output-mirror transmission coefficient on the gain and nonselective losses in a medium. In the West one remembers in this connection only the paper of Rigrod [11], although our work with similar calculations [12] was published somewhat earlier.

I also extended the multimode lasing model proposed by Tang and Statz [13,14] to the case of high lasing power [15, 16]. Although this analysis related to a quasi-stationary lasing regime which is practically never realized, nevertheless they permit one to estimate the limiting angular divergence of lasers with ideal plane resonators.

Nevertheless, plane resonators are employed most frequently in lasers containing media with substantial irregular inhomogeneities, and it is the latter that usually play a dominant role in the angular distribution. Russian literature has an enormous number of papers dealing with this problem, where one made use, as a rule, not of numerical but rather of various, and at times nonstandard, analytical approaches.

A weighty contribution here is due to V. V. Lyubimov. He was, in particular, the first to apply to resonators whose eigenfunctions do not formally form a complete set the perturbative approach [17]. This stimulated us to study more comprehensively the validity of this method as applied to plane resonators [18].

Qualitatively, the possibility of using an incomplete set of functions is based on the fact that in the case of resonators the matrix elements of the perturbation operator reflect the fraction of light which is scattered due to the perturbation from the original mode into others. For many kinds of perturbations, these fractions turn out to be significant only for a few modes close to the original one, which permits restricting oneself to taking into account only one or two terms in the expansion and, thus, avoid the messy summation of an infinite number of terms. In such cases this approach leads to results coinciding well with what is obtained in exact calculations.

Finally, already in the 80's I together with Anikichev have been able to dot the i's in the problem of expansion in eigenfunctions of open resonators [10].

Without going in further details, it can be maintained that all problems bearing on the angular divergence of lasers with stable and plane resonators, and those equivalent to them, were studied at that time to very fine details, and in some cases by invoking very original and efficient approaches. The time has now come to turn to unstable resonators.

5. A LITTLE ABOUT UNSTABLE RESONATORS

The role of Siegman is known to everybody here; his famous paper published in 1965 [19] provided an impetus to all this direction of research. At the same time nearly all laser devices based on unstable resonators and used in practice were proposed and investigated exhaustively in our country. This is explained by the fact that we started from the principle of unstable resonator design formulated in the most general form. It reads as follows: to convert a laser amplifier into a generator, a light beam taken from a part of the output-beam cross section is fed into the feedback system, and in the latter it is transformed in some way so as to fill all of the amplifier cross section.

Based on this principle, we developed various types of unstable resonators with a reduced sensitivity to misalignments [20-22], multipass telescopic amplifiers [23, 24], wide-aperture generators controlled by shutters or spectral selectors with a small cross section [25-27] , unstable resonator with a compact shape of the output aperture [28, 29] and so on. The only truly new idea in the area of unstable resonators which enjoyed certain applications was the idea of resonators with spatial filtration of radiation [30], which relates to systems with a small Fresnel number and was not considered by us.

6. ABOUT THERMAL DEFORMATIONS OF LASER RESONATORS

We are turning now to another essential factor which determines, together with the resonator, the angular divergence of radiation; I have in mind the optical inhomogeneity of the gain medium. I would say that the problem of optical inhomogeneity of the medium has been solved to some extent only for the narrow class of neodymium-glass lasers, where thermal effects provide the main source of inhomogeneities.

The first observations of thermally-induced strains were made on cryogenic fluorite lasers and were published in 1964 [31]. I have to confess that these experiments made with a Max-Zehnder interferometer and a non-laser light source were extremely time consuming. This setup was soon afterwards used to measure thermal deformations in neodymium-glass rods as well, but this did not result at the time in anywhere near constructive recommendations as to how to improve the situation. Such considerations were put forward in the very well-known paper by Snitzer [32], and they reduced to the following: by properly varying the glass composition one can reach noticeable mutual compensation of the changes in the refractive index associated with temperature variations and photoelastic effects.

Because the form of the dependence of the temperature and stresses on radial coordinate for a circular rod is different even under its axially symmetric heating, total compensation of the above factors is be achieved. Such a possibility appears, however, if one takes instead bars with an elongated rectangular cross section. The results of the corresponding analysis were published by us in 1970 [33]. We also introduced there the so-called laser thermo-optical constants which characterize refractive-index variations in rods of both circular and rectangular cross section and are widely used in Russian literature.

This stimulated a series of brilliant studies by our specialists in glass melting. They developed a wide range of glasses with unique thermo-optical characteristics (see, for instance, [34]).

At about the same time another way to combat thermo-optical effects in solid-state lasers was found. I have in mind the so-called slab lasers with zigzag ray propagation which enjoy presently wide recognition. The suggestion that thermo-optical effects should play a substantially smaller role in such lasers was put forward for the first time in 1968 [35] and realized in practice by Mikaelyan and D'yachenko in the early 70's [36]. Unfortunately, no further work was done along these lines in our country at that time, and these ideas became popular only in the 80's after the publications by Kane, Byer, Eggleston, and others [37, 38].

7. HISTORY OF INITIAL WORKS ON APPLICATION OF WAVE FRONT CONJUGATION FOR COMPENSATION OF GAIN MEDIUM INHOMOGENEITIES

I would like to say also a few words on another, the most universal method of combating the consequences of optical inhomogeneities, namely, the wave front conjugation. The essence of this method is well known, and I shall not dwell on it here. At the conference in Slovak Republic which I have already mentioned I met with a curious fact that although Western specialists mostly recognize the importance of Russian works in this area, they do not know the people who stimulated these studies.

One should name here in the first place V. V. Ragul'skii. It was he who developed the main physical concepts on the specific features of the mechanism of Mandelshtam-Brillouin stimulated scattering, which under certain conditions can result in wave front conjugation. The brilliant experiments aimed at realizing these conditions [39] initiated an avalanche of papers dealing with this effect and with the possibilities of its application.

It is remarkable, however, that the idea of the most effective application of wave front conjugation to designing high-power laser systems with close to diffraction-limited divergence in the presence of considerable optical inhomogeneities was conceived by Kiev rather than Moscow scientists. It should be pointed out that the former made a significant contribution to progress in other areas of quantum electronics as well; in particular, they published a pioneering series of papers on dispersive resonators and the theory of laser radiation spectra.

In late 60's, M. S. Soskin with coworkers (Kiev Institute of Physics) began to study the possibility of wave front correction with thin holograms [40]. The idea of their method in its simplest modification is illustrated by Fig. 2.

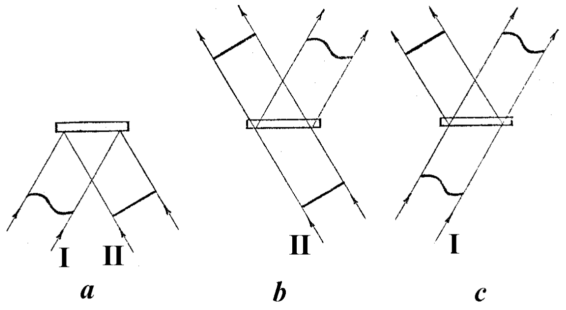

Fig. 2

The process of hologram recording is shown in Fig. 2,a. A thin hologram is recorded in interference of beam 2 with a plane wave front with beam 1 having a curved wave front. The latter in usual holography is a radiation scattered by one object; the reconstruction of its image is achieved when the ready hologram is illuminated by beam 2 (Fig. 2,b). If one illuminates this hologram by beam 1, the beam with a plane wave front similar to initial beam 1 is obtained (fig. 2, c); thus in this way one can in principle transform a high-power beam with a curved wave front into a beam with diffraction-limited divergence.

Among other things, this group of authors reported on successful demonstration of a reduction of angular divergence of ruby laser radiation. Of considerable interest is also their work of 1971 [41], where they reduced the divergence of a high-order mode in a stable resonator by eliminating the phase shift of π between adjacent spots. Western specialists learned of this idea from the paper of Casperson *et al* [42] published six years later.

Despite the obvious informative value of these works with thin holograms, there would not be any sense in trying to apply this method to correcting high-power laser radiation. There are two immediately evident reasons for this: the low radiation strength of thin holograms and the enormously high power losses involved because of the low diffraction efficiency characteristic of these holograms. Obviously enough, one can get rid of both these disadvantages by effecting the correction at the input rather than output of a high-power laser amplifier. As for the way in which one could obtain information on the wave front shape at amplifier input such that it becomes plane on passing through an non-uniform gain medium, it immediately suggests itself to make the beam propagate through the same medium but in the opposite direction, the process illustrated in Fig.3.

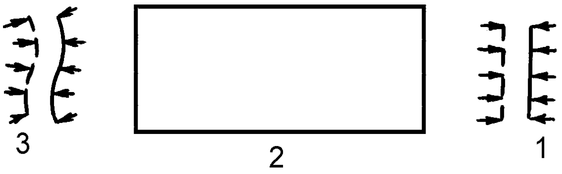

Fig. 3. Phase distortion compensation in laser amplifier:

1 - reference beam, 2 - amplifier, 3 - conjugate wave

This popular scheme of a two-pass amplifier with WFC was proposed by me in 1971 [43]. I was helped on my way to it, besides the above-mentioned works of the Kiev people, by an idea expressed by V. E. Sherstobitov on the possibility of achieving WFC by means of a spatial filter of the type of the Zernike cell. Another factor that helped me was that at that time I had been trying for several years to find a way of intracavity self-compensation for inhomogeneities of the gain medium, and had come to the conclusion that in generators with a large Fresnel number such attempts are doomed to failure.

8. CONCLUSION

In our large country there are many scientific schools which made a tremendous contribution to the development of research in this area. What I have just told you about was intended as just an illustration of this point using a few examples I know. It should be stressed that Russian studies, rather than repeating Western works, primarily complement them. This finds an easy explanation: indeed, we read the papers of Western scientists and, quite naturally, tried to do something new. Most of this “new” can be found in my book [1], which sums up, besides Western works, the results of a few hundred Russian publications. One can find here quite a number of promising ideas which, similar to the slab laser, were not put in practice because of the inferior state of our technology at the time.

If we add all the knowledge accumulated in this area, we shall see that there are, alas, very few “white spots” left in the science of resonators and the problem of divergence of laser radiation. The number of new bright ideas appearing in this domain has dropped dramatically. Nevertheless, there still remain a few interesting special problems. Just as an illustration, consider resonators for lasers with an annular cross section of the gain medium. Although numerous methods of solving this problem have already been proposed and keep being put forward, among them are no simple and, at the same time, efficient approaches. I shall permit myself mentioning here two techniques which, in my opinion, deserve attention.

One of them is based on using a specific astigmatic telescope, whose starting idea was proposed by my disciple Anikichev. It has already been discussed in a few publications (see, e.g., [44, 45]), and therefore I shall not dwell on it here, but want to stress only its considerable potential. Another interesting possibility of making radiation of a coaxial waveguide laser coherent in azimuthal direction is to use the well-known Talbott effect. It can be shown that this effect occurs not only when the grating and the screen are plane but also when their surfaces are opposite parts of the side surface of a circular cylinder. This permits one to increase the coupling between different parts of the waveguide cross section using the technique illustrated by Fig. 4. The beam with a plane wave front and a narrow annular cross section exiting coaxial waveguide 1 strikes mirror 2 whose surface is close to that of a cone with the angle at the vertex of 90°. The slight curvature of the generating lines acts so as to compress the beam cross section to minimize the losses introduced when inputting the radiation back into the waveguide. A periodic system of grooves $\lambda/2\sqrt{2}$ deep is cut along the generating lines to produce a phase shift of π between the radiation reflected from the groove bottom and from the remaining parts of the mirror. The period along the azimuthal direction should be such as to make the average waveguide diameter D a multiple of the Talbott length. After the second reflection from this mirror, the phase shifts double to make 2π, and in this way a plane wave front is reconstructed.

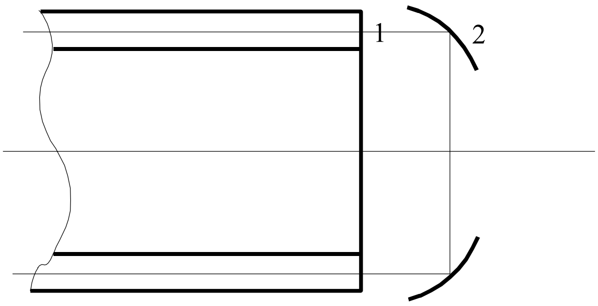

Fig. 4

It is essential that realizing the Talbott effect outside a coaxial waveguide provides necessary conditions for the existence inside the waveguide of only one mode with zero indices for both radial and azimuthal directions. And with this amusing support for the suggestion that the store of new ideas has not yet exhausted to the bottom I shall finish my message.

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